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ON THE LOGICAL DEFINABILITY OF CERTAIN GRAPH AND POSET LANGUAGES

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ABSTRACT

We show that it is equivalent, for certain sets of finite graphs, to be definable in *CMS* (counting monadic second-order logic, a natural extension of monadic second-order logic), and to be recognizable in an algebraic framework induced by the notion of modular decomposition of a finite graph.

More precisely, we consider the set $\mathcal{F}_\infty$ of composition operations on graphs which occur in the modular decomposition of finite graphs. If $\mathcal{F}$ is a subset of $\mathcal{F}_\infty$, we say that a graph is an $\mathcal{F}$-graph if it can be decomposed using only operations in $\mathcal{F}$. A set of $\mathcal{F}$-graphs is recognizable if it is a union of classes in a finite-index equivalence relation which is preserved by the operations in $\mathcal{F}$. We show that if $\mathcal{F}$ is finite and its elements enjoy only a limited amount of commutativity – a property which we call weak rigidity, then recognizability is equivalent to *CMS*-definability. This requirement is weak enough to be satisfied whenever all $\mathcal{F}$-graphs are posets, that is, transitive dags. In particular, our result generalizes Kuske's recent result on series-parallel poset languages.

Keywords: Graph languages, logical definability, algebraic recognizability

1. Introduction

The connection between recognizability and definability is one of the cornerstones of theoretical computer science, going back to Büchi's celebrated theorem on finite and infinite words in the 1960s (see [26]). This theorem states the equivalence between two fundamental properties of a language:

- to be definable in monadic second order logic (*MS*),
- to be recognizable.

In Büchi's work, recognizability is defined by means of a finite state automaton. It is well-known that recognizability by such an automaton is equivalent to *algebraic recognizability*, that is, to being the union of classes in a finite-index congruence.

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