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SPANNING 2-CONNECTED SUBGRAPHS IN ALPHABET GRAPHS, SPECIAL CLASSES OF GRID GRAPHS [1]

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ABSTRACT

A grid graph G is a finite induced subgraph of the infinite 2-dimensional grid defined by $Z \times Z$ and all edges between pairs of vertices from $Z \times Z$ at Euclidean distance precisely 1. A natural drawing of G is obtained by drawing its vertices in $\Re^2$ according to their coordinates. Apart from the outer face, all (inner) faces with area exceeding one (not bounded by a 4-cycle) in a natural drawing of G are called the holes of G. We define 26 classes of grid graphs called alphabet graphs, with no or a few holes. We determine which of the alphabet graphs contain a Hamilton cycle, i. e. a cycle containing all vertices, and solve the problem of determining a spanning 2-connected subgraph with as few edges as possible for all alphabet graphs.

Keywords: Alphabet graph, grid graph, Hamilton cycle, spanning 2-connected subgraph

1. Introduction

The *infinite grid graph* G^∞ is defined by the set of vertices $V = \{(x,y) \mid x \in Z, y \in Z\}$ and the set of edges E between all pairs of vertices from V at Euclidean distance precisely 1. For any integers $s \geq 1$ and $t \geq 1$, the *rectangular grid graph* $R(s,t)$ is the (finite) subgraph of G^∞ induced by $V(s,t) = \{(x,y) \mid 1 \leq x \leq s, 1 \leq y \leq t, x \in Z, y \in Z\}$ (and just containing all edges from G^∞ between pairs of vertices from $V(s,t)$). This graph $R(s,t)$ is also known as the *product graph* $P_s \times P_t$ of two disjoint paths P_s and P_t. A *grid graph* is a graph that is isomorphic to a subgraph of $R(s,t)$ induced by a subset of $V(s,t)$ for some integers $s \geq 1$ and $t \geq 1$.

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