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A SMALL EMBEDDING FOR PARTIAL 4-CYCLE SYSTEMS WHEN THE LEAVE IS SMALL [1]

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ABSTRACT

In this paper we give an embedding for odd n which improves the best known bound when the "leave" is small. In particular, we prove that a partial 4-cycle system of odd order n with leave consisting of x edges can be embedded in a 4-cycle system of order $n+2x$.

Keywords: Embedding, 4-cycle systems, bound, partial 4-cycle systems

1. Introduction

A 4-cycle system of order n is a pair (S, C), where C is a collection of edge-disjoint 4-cycles which partitions the edge set of the complete undirected graph K_n with vertex set S. It is well-known that the spectrum for 4-cycle systems (= the set of all n such that a 4-cycle system of order n exists) is precisely the set of all $n \equiv 1 \pmod 8$. (See for example [3].)

A *partial* 4-cycle system of order n is a pair (X, P), where P is a collection of edge-disjoint 4-cycles of the edge set of K_n with vertex set X. The difference between a partial 4-cycle system and a 4-cycle system is that the edge-disjoint 4-cycles belonging to a partial 4-cycle system do not necessarily include all of the edges of K_n.

A natural question to ask is the following: given a partial 4-cycle system (X, P) of order n, is it always possible to decompose $E(K_n) \setminus E(P)$ into edge-disjoint 4-cycles? ($E(K_n) \setminus E(P)$ = the complement of the edge set of P in the edge set of K_n.) That is, can a partial 4-cycle system always be *completed* to a 4-cycle system? The answer to this question is no, since any partial 4-cycle system of order $n \not\equiv 1 \pmod 8$ (and most of the ones that are) cannot be completed.

So the following problem is of interest. Can we always *embed* a partial 4-cycle system in some 4-cycle system? The partial 4-cycle system (X, P) is said to be *embedded* in the 4-cycle system (S, C) provided $X \subseteq S$ and $P \subseteq C$. Naturally, we would like the size of the containing system to be as small as possible.

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