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LATIN TRADE ALGORITHMS AND THE SMALLEST CRITICAL SET IN A LATIN SQUARE [1]

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ABSTRACT

A critical set is a partial Latin square that has a unique completion to a Latin square, and is minimal in this property. Suppose that P is a critical set in a Latin square L of order n, and there is one row of P which is empty. Then there are at most two rows of P with precisely one entry, and thus $|P| \geq 2n - 4$. Moreover, in this case these three rows in L are isotopic to three adjacent rows in the back circulant Latin square. In our proof new algorithms for constructing Latin trades in arbitrary Latin squares are given.

Keywords: Latin trade, critical set, Latin square, circulant Latin square

1. Introduction

In any combinatorial configuration it is possible to identify a subset which uniquely determines the structure of the configuration and in some cases is minimal with respect to this property. Examples of such subsets can be found by studying the literature on critical sets in Latin squares (see Donovan and Howse [8]) and defining sets in block designs (see Street [11]). The recent research in these areas has focused on building a bank of knowledge which may be used to determine the spectrum of the prescribed subsets. With this current paper we restrict ourselves to a discussion of critical sets in Latin squares.

We define $\mathrm{scs}(n)$ to be the size of the smallest critical set in any Latin square of order n. The problem of determining this value exactly for every n remains unsolved. However, progress has been made on upper and lower bounds.

Fu, Fu and Rodger [9] showed that if $n > 20$, $\mathrm{scs}(n) \geq \lfloor (7n-3)/6 \rfloor$. This bound was recently improved by Horak, Aldred and Fleischner [10], who showed that if $n \geq 8$, $\mathrm{scs}(n) \geq \lfloor (4n-8)/3 \rfloor$. The smallest critical set so far constructed for any Latin square of size n has size $\lfloor n^2/4 \rfloor$ (see [6, 5]). A critical set of such size is known to exist in back circulant Latin squares, namely those Latin squares based on the addition table for the integers modulo n.

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