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LANGUAGES RELATED TO THE PROPERTIES OF DISJUNCTIVITY AND CODE

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ABSTRACT

In this paper, we define two new types of words, d_1-words and d_2-words and show that the free semigroup X^+ can be represented as a disjoint union of the disjunctive languages $D(1)$, the set of all d-primitive words, N_{d_1} the set of all d_1-words and N_{d_2} the set of all d_2-words. Both the languages N_{d_1} and N_{d_2} are also shown to be disjunctive languages. We show that for any language $L \subseteq X^+$, the language $L \cdot Q$, where Q is the set of all primitive words, is not a 2-code and we give a characterization of that $P \cdot Q^{(i)}$ is a 2-code for some prefix code P and $Q^{(i)} = \{f^i \mid f \in Q\}$, $i \geq 2$. We proved that each of the disjunctive languages Q, $D(1)^{(i)}$, $i \geq 1$, $D(n)$, $n \geq 2$, N_{d_1}, and N_{d_2} removing a code from it results a dense language. On the other hand, the set $Q^{(i)}$, $i \geq 2$ removing a code or Q removing a prefix code both results disjunctive languages. Any disjunctive language union a non-dense language is disjunctive. Also any disjunctive language removing a non-dense language is also shown to be disjunctive.

Keywords: Disjunctive language, prefix code, 2-code, dense language, d_1-word, d_2-word

1. Introduction

In this paper we assume that X is a finite alphabet containing more than one letter. Let X^* be the free monoid generated by X. Any element of X^* is a *word* and any subset of X^* is a *language*. For any language L, let $|L|$ be the cardinality of L. For any two languages L_1, L_2 contained in X^*, by $L_1 \subseteq L_2$, we mean that L_1 is a subset of L_2. We denote it by $L_1 \subset L_2$ when $L_1 \subseteq L_2$ and $L_1 \neq L_2$. Let $X^+ = X^* \setminus \{1\}$, where 1 is the empty word. For any two languages A and B, the concatenation of A and B is the set $AB = \{xy \mid x \in A,\ y \in B\}$ and $A^+ = A \cup A^2 \cup A^3 \cup \cdots$. For a word $w \in X^*$, let $\lg(w)$ be the length of the word w, which is just the total number