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DETERMINIZATION OF FINITE STATE WEIGHTED TREE AUTOMATA

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ABSTRACT

We investigate the determinization of nondeterministic bottom-up/top-down finite state weighted tree automata over some semiring A and compare the resulting four classes of formal tree series with each other. In particular, for every bottom-up finite state weighted tree automaton over some semifield (i. e., a semiring which contains a multiplicative inverse for every nonzero element) we construct a deterministic bottom-up weighted tree automaton. This construction is called partial determinization, because it may lead to an automaton with an infinite set of states. We prove sufficient conditions under which the partial determinization produces a bottom-up *finite state* weighted tree automaton which is equivalent to the original one. Using this partial determinization we generalize well known theorems on classes of tree languages (cf. [14] Chapter II, Theorems 2.6 and 2.10, Example 2.11), viz if A is a commutative and locally finite semifield, then (i) nondeterministic bottom-up, (ii) deterministic bottom-up, and (iii) nondeterministic top-down finite state weighted tree automata are equally powerful. Moreover, if the input alphabet is not trivial, then deterministic top-down finite state weighted tree automata are strictly less powerful than the aforementioned classes.

Keywords: Weighted tree automata, determinization, recognizable tree series

1. Introduction

Finite state weighted tree automata have been investigated in, e. g., [10, 18, 24]. Conceptually, they are obtained from finite state tree automata by replacing the Boolean semiring by an arbitrary semiring. Let us first make some remarks on finite state tree automata and the classes of tree languages accepted by them, before we discuss the concept of a determinization algorithm and its instances for various automata models.

The concept of finite state tree automata is a generalization of the concept of finite state (string) automata in the sense that tree automata accept trees rather than strings. According to the way in which a given input tree t is traversed, there are bottom-up tree automata [8, 25] (or: frontier-to-root automata) in which t is traversed by starting from its leaves and proceeding towards the root, and top-down tree automata [19, 21] (or: root-to-frontier automata) in which t is traversed by starting

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