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WEIGHTED FINITE AUTOMATA AND METRICS IN CANTOR SPACE[1]

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ABSTRACT

We show how weighted finite automata define topologies on the set of all ω-words over a finite alphabet X. Moreover, we give a characterization of these topologies in terms of topologies on X^ω induced by languages $U \subseteq X^*$.

Keywords: Weighted finite automata, ω-words, topologies

1. Introduction

Weighted finite automata are used to describe fractal images (cf. [1, 2]). In particular, they play some rôle in the computation of the contraction coefficients for so-called Multiple Recursive Function Systems (MRFS are a combinations of finite automata with iterated function systems). This is explained in more detail in [6].

Here in a finite automaton $\mathcal{A} = (X, (0,\infty), Z, z_0, f, g)$ with output monoid $((0,\infty),\cdot)$ to each transition the contraction coefficient of the mapping φ_x corresponding to the input letter $x \in X$ is assigned. The resulting output $g(s,w)$ is an upper bound to the contraction coefficient of the mapping $\varphi_w := \varphi_{x_1} \circ \cdots \circ \varphi_{x_n}$ ($w = x_1 \ldots x_2$).

In case the contraction coefficients of φ_w for $w \to \xi \in X^\omega$ converge to zero the corresponding MRFS "draws a point for $\xi \in X^\omega$". The set of all ξ can be described topologically by a suitable topology (depending on the automaton $\mathcal{A}$ [6]).

Another kind of topology on X^ω are the U-δ-topologies introduced in [8] (for related topologies see also [5]). Here the distance between two ω-words $\xi, \eta \in X^\omega$ depends of the number of their common prefixes in the given language $U \subseteq X^\omega$.

In the present paper we give a relationship between both topologies. It turns out that every automaton-definable topology is a U-δ-topology for a suitable $U \subseteq X^*$.

A construction for $U \subseteq X^*$ from a given automaton is described. Conversely, we derive a property of automaton-definable topologies which proves that not every U-δ-topology can be defined by a weighted automaton. A last result shows that for every regular language $U \subseteq X^*$ the corresponding U-δ-topology is definable by a WFA.

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