

## COINDUCTIVE COUNTING WITH WEIGHTED AUTOMATA <sup>1</sup>

J. J. M. M. RUTTEN

*CWI, P. O. Box 94079, NL-1090 GB Amsterdam, The Netherlands*<sup>2</sup>  
*e-mail: janr@cwi.nl*

### ABSTRACT

A general methodology is developed to compute the solution of a wide variety of basic counting problems in a uniform way: (1) the objects to be counted are enumerated by means of an infinite weighted automaton; (2) the automaton is reduced by means of the quantitative notion of stream bisimulation; (3) the reduced automaton is used to compute an expression (in terms of stream constants and operators) that represents the stream of all counts.

*Keywords:* Coinduction, coalgebra, stream derivative, counting, weighted automaton

### 1. Motivation

There is a trend in combinatorial analysis started by several schools around 1980, where much emphasis is placed on the relations between combinatorial structure and the algebraic structure of generating functions. Over the years, this has led to several formalizations (addressing the foundational question: what *is* a combinatorial structure? How do we “specify” it? What is the relation of such specifications to counting?) being introduced by Goulden-Jackson [7], Flajolet-Sedgewick [5, 6], Joyal [1], Stanley [15, 16], and several others. Here we add one more formal system to the list, called the method of coinductive counting. From the enumerative point of view, it makes it possible to derive existing counting results in a new perspective. We first illustrate the method of coinductive counting by means of an example, and then summarize motivation and contents of the paper.

The following counting problem is taken from [8, p. 291]. Male bees are called drones and female bees are called queens. Drones are born out of a queen and have no father; a queen is born out of a father drone and a mother queen. The first few

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<sup>2</sup>Also: Faculty of Mathematics and Computer Science, Vrije Universiteit, De Boelelaan 1081a, NL-1081 HV Amsterdam, The Netherlands.