

FORMAL TREE SERIES¹

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ABSTRACT

In this survey we report on generalizations of some results on formal tree languages, tree grammars and tree automata. These generalizations are achieved by an algebraic treatment using semirings, fixed point theory, formal tree series and matrices. By the use of these mathematical constructs definitions, constructions, and proofs are obtained that are very satisfactory from a mathematical point of view. The main surveyed results of the paper are the following:

1. Tree automata (resp. finite, polynomial tree automata), whose behaviours are tree series over a semiring, and systems of equations (resp. finite, polynomial systems of equations), whose least solutions are tuples of tree series over a semiring, are equivalent.

2. A Kleene result: the class of recognizable tree series is characterized by rational tree series expressions.

3. Pushdown tree automata, whose behaviours are tree series over a semiring, and algebraic tree systems are equivalent; moreover, the class of algebraic tree series is characterized by algebraic tree series expressions (a Kleene result).

4. The class of recognizable tree series is closed under nondeterministic simple recognizable tree series transductions.

5. The families of recognizable tree series and of algebraic tree series are full abstract families of tree series (full AFTs).

6. The macro power series, a generalization of the indexed languages, and the algebraic power series are exactly the yields of algebraic tree series and of recognizable tree series, respectively; there is a Kleene result for macro power series; the yield of a full AFT is a full abstract family of power series.

Keywords: Semirings, tree automata, recognizable and algebraic tree series, full abstract families of tree series

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