

## AN EXTENSION THEOREM WITH AN APPLICATION TO FORMAL TREE SERIES<sup>1</sup>

STEPHEN L. BLOOM<sup>2</sup>

*Department of Computer Science, Stevens Institute of Technology  
Hoboken, NJ 07030, USA*

*e-mail: bloom@cs.stevens-tech.edu*

and

ZOLTÁN ÉSIK<sup>3</sup>

*Institute for Informatics, University of Szeged  
Szeged, Hungary*

*e-mail: esik@sol.cc.u-szeged.hu*

### ABSTRACT

Suppose that  $T$  is a grove theory, and  $M$  is a Conway or iteration matrix theory which is a subtheory of  $T$ . We provide a sufficient condition under which the dagger operation on  $M$  can be extended to  $T$  so that  $T$  becomes a Conway or iteration theory. Moreover, we prove a general Kleene type theorem applicable to all grove theories which are Conway theories. We discuss applications to synchronization trees and formal tree series.

*Keywords:* Iteration theory, grove theory, formal tree series

### 1. Introduction

In many areas of theoretical and applied computer science, one is interested in solving systems of fixed point equations

$$\begin{aligned}x_1 &= t_1(x_1, \dots, x_n, y_1, \dots, y_p) \\ &\vdots \\ x_n &= t_n(x_1, \dots, x_n, y_1, \dots, y_p).\end{aligned}\tag{1}$$

where, for each  $i \in [n]$ ,  $t_i$  is a function  $t_i : A^{n+p} \rightarrow A$ , for some structure  $A$ . The structure may be a poset of some kind, and the functions may be order-preserving or

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