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ON THE PALINDROMIC DECOMPOSITION OF BINARY WORDS

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ABSTRACT

We prove a precise formula for the minimal number $K(n)$ such that every binary word of length n can be divided into $K(n)$ palindromes. Also we estimate the average number $\overline{K}(n)$ of palindromes composing a random binary word of the length n.

Keywords: Binary word, palindrome, measure of symmetry, measure of asymmetry

1. Introduction

The present note arose from the following problem proposed at International Mathematical Tournament of Towns [4, p. 8]:

> *Prove that every binary word of length* 60 *can be divided into* 24 *symmetric subwords and that the number* 24 *cannot be replaced by* 14.

A word $w = a_0 \dots a_n$ is called *symmetric* if $a_i = a_{n-i}$ for all $i \leq n$. For symmetric words we shall use a more poetic term *palindrome.* Let S be the set of nonempty binary words over the alphabet $\{a, b\}$ and S^1 be the set S with added the empty word. Observe that the set S^1 is a semigroup with respect to the operation of concatenation. The length of a word $w \in S^1$ will be denoted by $l(w)$. In particular, the empty word has length 0.

The above tournament task suggests three general problems:

(1) *Given a word $w \in S$ find the minimal number $m(w)$ of palindromes whose product in S is equal to w (thus the number $m(w)$ can be thought as a measure of asymmetry of w);*

(2) *Given a positive integer n find the number $K(n) = \max\{m(w) : l(w) = n\}$ equal to the maximal asymmetry measure of the "worst" binary word of length n;*

(3) *Estimate the average asymmetry measure $\overline{K}(n) = 2^{-n} \sum\{m(w) : l(w) = n\}$ of a random binary word of length n.*