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# A NEW MEASURE OF ASYMMETRY OF BINARY WORDS

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ABSTRACT

A binary word is symmetric if it is a palindrome or an antipalindrome. We define a new measure of asymmetry of a binary word equal to the minimal number of letters of the word whose deleting from the word yields a symmetric word and obtain upper and lower estimations of this measure.

*Keywords:* Palindrome, measure of symmetry, measure of asymmetry

In this paper under the term word we shall understood the binary word over the alphabet $\{a, b\}$. The set of all words can naturally be thought as a free semigroup with two generators, which we shall use in the notation of the words. There are various approaches to a measure of asymmetry of a word or a group coloured into two colours [2, 4] (see [1] for more references). In the present paper we investigate a new measure of symmetry of a word, suggested by the question of Ihor Protasov [3]. By a *symmetric word* we shall understood a word which is a palindrome or an antipalindrome. A word $w = a_1 \ldots a_n$ is called a *palindrome* (respectively, an *antipalindrome*) if $a_i = a_{n-i+1}$ (respectively, $a_i \neq a_{n-i+1}$) for all $i \leq n$. Given a word $w$ let $S_d(w)$ be the minimal number of letters of $w$ whose deleting from $w$ yields a palindrome or an antipalindrome. Observe that a word $w$ is symmetric if and only if $S_d(w) = 0$. Thus the number $S_d(w)$ can be thought as an asymmetry measure of $w$. For every positive integer $n$ let $S_d(n) = \max\{S_d(w) : w \in S,\ l(w) = n\}$ be the maximal asymmetry measure $S_d(w)$ of a word $w$ of length $l(w) = n$.

Ihor Protasov observed that $S_d(n) \leq n/3$ for small $n$ and asked in [3] if this estimation holds for every $n$. Computer calculations show that this conjecture fails already for $n = 10$. The values of $S_d(n)$ for $n \leq 20$ are given in Table 1.

The main result of the paper is the following theorem.

**Theorem 1** *For every $n \geq 2$ the number $S_d(n)$ lies in the range*

$$\left[\frac{n + 2\left[\frac{n-3}{7}\right]}{3}\right] \leq S_d(n) \leq \left[\frac{n}{2}\right].$$