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ON THE COMPLEXITY OF A PROBLEM ON MONADIC STRING REWRITING SYSTEMS [1]

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ABSTRACT

Computing the set of descendants of a regular language L with respect to a monadic string rewriting system has proved to be very useful in developing decision algorithms for various problems on finitely presented monoids and context-free grammars. Recently, Esparza et al. [7] proved $\mathcal{O}(ps^3)$ time and $\mathcal{O}(ps^2)$ space bounds for this problem, where p is the number of rules in the monadic string rewriting system and s is the number of states in the automaton accepting L.

Using *synchronized extension systems* [10, 11, 12] we provide a new insight into the problem and present an $\mathcal{O}(pr)$ time and space solution, where p is as above and r is the number of rules in the grammar generating L.

Keywords: Formal languages, decision problems, monadic string rewriting systems, synchronized extension systems, computational complexity

1. Introduction and Preliminaries

In [4] (see also [3]), Book and Otto show, among many other results, that if the rewriting rules of a monadic string rewriting system T are applied to the strings of a regular set L, the set $\Delta_T^*(L)$ so obtained (the set of descendants of L with respect to T) is also regular. This result can be used to develop a new approach for designing decision algorithms for various problems on finitely presented monoids and

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