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ABOUT SOME OVERLAP-FREE MORPHISMS ON A n-LETTER ALPHABET [1]

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ABSTRACT

In 1912, the Norwegian mathematician Axel Thue was the first to describe an overlap-free binary infinite word. This word was generated by a morphism which is called, since the works of Morse, the Thue-Morse morphism.

Here we study morphisms, generalizing the Thue-Morse morphism in the case of alphabets with more than two letters, which are obtained from a construction made by Prouhet in 1851. We examine in which case these morphisms are overlap-free and prove that the Prouhet words they generate are rigid. We also give a link with the construction realized by Arshon in 1937, proving in particular that the n-letter Arshon word is generated by morphism if and only if n is an even number. These words are also rigid.

Keywords: Overlap-free morphisms and words, Thue-Morse morphism and word, Prouhet words, Arshon words, rigid words

1. Introduction

In 1906, Thue was the first to publish a paper [29] in which were explicitly studied the combinatorial properties of strings of letters. In 1912, he published another paper [30] in which was in particular introduced a binary infinite overlap-free word, now called the *Thue-Morse word*. Thue's results were rediscovered independently years after (see Hedlund, 1967 [12] and, to know what Thue exactly did, Berstel, 1992 [5]). In particular Morse [20] proved again that the Thue-Morse word $\mathbf{t}$ is overlap-free. This word is generated by the *Thue-Morse morphism* μ defined on the alphabet $A = \{0, 1\}$ by $\mu(0) = 01$, $\mu(1) = 10$. Thus $\mathbf{t} = 0110100110010110100101100110100110\ldots$ It also appears as a special case of what Adler and Li [1] called *Prouhet sequences* (see Prouhet, 1851 [23]). Actually Prouhet gave an algorithm to realize an arithmetic construction (a solution to the so-called Tarry-Escott problem, see Lehmer, 1947 [15], Hardy and Wright, 1985 [10]); this algorithm produces intermediate infinite words, and it happens that on the two-letter alphabet this word is the Thue-Morse one. Also, these words are generated by morphisms and, in the two-letter case, the morphism is μ.

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