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DESCRIPTIONAL COMPLEXITY OF CELLULAR AUTOMATA AND DECIDABILITY QUESTIONS[1]

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ABSTRACT

We study the descriptional complexity of cellular automata (CA) which are a parallel model of computation. We show that between one of the simplest cellular models, the realtime one-way CA (realtime-OCA), and "classical" models like deterministic finite automata or pushdown automata, there will be savings concerning the size of description not bounded by any recursive function, so-called nonrecursive trade-offs. Furthermore, nonrecursive trade-offs are shown to exist between certain restricted classes of cellular automata. The set of valid computations of a Turing machine can be recognized by a realtime-OCA. This implies that many decidability questions are not even semidecidable for cellular automata. There is no pumping lemma and no minimization algorithm for cellular automata. Finally, we prove that the language class accepted by realtime-OCA is incomparable to many known and well-investigated language classes.

Keywords: Cellular automata, descriptional complexity, decidability questions, nonrecursive trade-offs

1. Introduction

Given a grammar or automata model, in the theory of formal languages one investigates for example the generative capacity, closure properties or decidability questions of the model. Furthermore, questions concerning the descriptional complexity arise. How succinctly can a model represent a formal language in comparison with other models? Regarding regular languages, it is known [12] that there is an infinite sequence of languages $(L_n)_{n\in\mathbb{N}}$ such that each language L_n is recognized by a nondeterministic finite automaton (NFA) with n states, but every deterministic FA (DFA) recognizing L_n will need 2^n states. Beyond this trade-off bounded by an exponential function, Hartmanis has proven in [6] that between pushdown automata (PDA) and deterministic PDA (DPDA) there exists a trade-off not bounded by any recursive function, a so-called nonrecursive trade-off. Additional nonrecursive trade-offs are known to exist between unambiguous PDA (UPDA) and DPDA, between PDA and UPDA and many other models.

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