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REFINING NONDETERMINISM BELOW LINEAR TIME[1]

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ABSTRACT

Multitape Turing machines with a restricted number of nondeterministic steps are investigated. Fischer and Kintala showed an infinite nondeterministic hierarchy of properly included real-time languages between the deterministic languages and the log-bounded nondeterministic languages. This result is extended to time complexities in the range between real time and linear time, and is generalized to arbitrary dimensions.

For fixed amounts of nondeterminism infinite proper dimension hierarchies are presented. The hierarchy results are established by counting arguments. For an equivalence relation and a family of witness languages the number of induced equivalence classes is compared to the number of equivalence classes distinguishable by the model in question. By contradiction the properness of the inclusions is proved.

Keywords: Limited nondeterminism, multitape Turing machines, nondeterministic hierarchies, dimension hierarchies, fast computations

1. Introduction

One of the central questions in complexity theory asks for the power of nondeterminism in bounded-resource computations. Traditionally, nondeterministic devices have been viewed as having as many nondeterministic guesses as time steps. The studies of this concept of unlimited nondeterminism led, for example, to the famous open LBA-problem or the unsolved question whether or not P equals NP. In order to gain further understanding of the nature of nondeterminism, in [4, 11] it has been viewed as an additional limited resource at the disposal of time or space bounded computations.

Motivated by the search for problems that are neither in P nor NP-complete the same authors investigated the so-called β-hierarchy [12]. They considered languages acceptable by polynomial-time bounded Turing machines that make a polylogarithmic number of nondeterministic steps. The hierarchy relies on the exponent of the logarithm. For every $c \in \mathbb{N}$ the class of languages acceptable with $O(\log^c)$ nondeterministic steps is included in the class acceptable with $O(\log^{c+1})$ nondeterministic steps. Clearly, since the hierarchy is in between P and NP none of the inclusions is

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