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STATE COMPLEXITY OF PROPORTIONAL REMOVALS [1]

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ABSTRACT

We examine the state complexity of proportional removals such as $\frac{1}{2}(L)$. For $\frac{1}{2}(L)$, we show a bound which is tight in the case that L is a unary language, and an nearly optimal bound for arbitrary languages. We also compute the average state complexity for $\frac{1}{2}(L)$ if L is unary. We study other proportional removals and give bounds for certain reset automata.

Keywords: State complexity, reset automata, proportional removals

1. Introduction and Motivation

The *state complexity* of a regular language L is the number of states in the minimal deterministic finite automaton (DFA) recognizing L. There has been much interest lately in the study of state complexity of operations which preserve regularity (e. g. [2, 9]). These papers are generally interested in proving upper bounds on the state complexity of operations on regular languages, including the particular case of unary regular languages. In this paper, we examine proportional removals, that is, languages of the form

$$\{x \in \Sigma^* \mid \exists y \in \Sigma^* \text{ such that } r(|x|, |y|) \wedge xy \in L\} \tag{1}$$

for some language $L \subseteq \Sigma^*$, and a binary relation r. There is a complete characterization, due to Seiferas and McNaughton [6], of the relations r which ensure that (1) is a regular language, if L is regular. We obtain bounds for the equality relation, for both unary and general languages, and show this bound is tight for unary languages.

2. Notation and Definitions

We assume the reader is familiar with basic concepts in automata theory and formal languages. For any unfamiliar terminology, see Hopcroft and Ullman [4] or S. Yu [8].

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