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LINEAR LANGUAGES WITH A NONASSOCIATIVE CONCATENATION

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ABSTRACT

We introduce a new Kleene star operation. Linear languages with a nonassociative concatenation can then be denoted by regular-like expressions using this Kleene star operation. The proposed method can also be applied to linear languages with the usual associative concatenation.

Keywords: Formal languages, linear languages, regular expressions

1. Introduction

The formal language theory is a tool for modelling the syntax of natural languages. It has been developed mainly over free monoids generated by alphabets, namely over sets with an associative concatenation. It should be interesting to drop this requirement of associativity. The main motivation for eliminating the associativity assumption stems for linguistic considerations. In the word-sequence of a sentence, concatenation is semantically nonassociative: see [11, p. 157] for details. Our approach can be considered as a part of the topical trend in mathematics and theoretical physics, two fields which include methods of nonassociative algebras in their arsenals more and more frequently. See for example the recent book's chapter "Nonassociative structures" in [13] and the special issue on nonassociative algebras in [14]. Why not using the same approach in formal languages theory?

A natural way to grasp nonassociativity lies in considering trees instead of words. This has sparked a large number of papers on tree grammars and tree languages [3]. The study of tree languages has later been emphasized by a specific tool: formal power series on trees, which was initiated by Berstel and Reutenauer [5] and carried forward by Bozapalidis [1, 7, 8]. Formal power series on trees appear as a generalization of the classical theory of formal power series on words [6, 20].

In a more algebraic approach [16, 17, 19], it has been shown that the associativity of the concatenation α can be dropped by embedding α into a denumerable set of binary operations $\{\alpha_p\}_{p\in\mathbb{Z}}$ satisfying the following property: $\alpha_p(x, \alpha_q(y, z)) =$