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CHARACTERIZATIONS OF CERTAIN MATROIDS VIA FLATS

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ABSTRACT

The aim of this paper is to discuss properties of open-regular-flats, open-feeble-flats, alternative-sets and inner-flats. Moreover, we characterize some peculiar matroids via these notions. Finally, we provide a new decomposition of strong maps.

Keywords: Matroid, strong map, hesitant map, feeble-open-set, open-regular-flat, alternative-set, inner-flat, open-feeble-flat

1. Introduction

The matroid terminology and notation will follow [7]. For an introduction on matroids, see also [3, 4, 5, 6]. In particular, a *matroid* M is an ordered pair $(E, \mathcal{O})$ such that $\mathcal{O}$ is a collection of subsets, called *open sets* of M, of a finite set E, called the *ground set* of M, such that $\varnothing$ is an open set, unions of open sets are open and if O_1 and O_2 are open sets and $x \in O_1 \cap O_2$, there exists an open set O_3 such that

$$(O_1 \cup O_2) - (O_1 \cap O_2) \subseteq O_3 \subseteq (O_1 \cup O_2) - \{x\}.$$

An equivalent way of defining a matroid M, is that M is an ordered pair $(E, \mathcal{F}_M)$ such that $\mathcal{F}_M$ is a collection of subsets, called *flats* or *closed sets* of M, of a finite set E such that $E \in \mathcal{F}_M$, intersections of flats are flats and if $F \in \mathcal{F}_M$ and $\{F_1, F_2, \ldots, F_k\}$ is the set of minimal members of $\mathcal{F}_M$ (with respect to inclusion) that properly contain F, then $F_1 \cup F_2 \cup \cdots \cup F_k = E$. Clearly, a subset $A \subseteq E$ is a flat if and only if its complement $E \setminus A$ is an open set. The *closure* of a subset $A \subseteq E$ will be denoted by $\overline{A}$. Clearly $\overline{A}$ is the smallest flat containing A and $x \in \overline{A}$ if and only if for every open set O in M that contains x, $O \cap A \neq \varnothing$, see [4].

Let $M_1 = (E_1, \mathcal{F}_1)$ and $M_2 = (E_2, \mathcal{F}_2)$ be matroids. A *strong map* f from M_1 to M_2 is a map $f : E_1 \to E_2$ such that the inverse image of any flat of M_2 is a flat of M_1. We abbreviate this as $f : M_1 \to M_2$. Clearly, f is strong if and only if the inverse image of any open set in M_2 is open in M_1. A set $U \subseteq E$ is called a *feeble-open-set* in $M = (E, \mathcal{O})$ if there exists an open set $O \in \mathcal{O}$ such that $O \subseteq U \subseteq \overline{O}$. A subset $A \subseteq E$ is *feeble-flat* if its complement is feeble-open. The *inner* of A is the