Journal of Automata, Languages and Combinatorics **7** (2002) 2, 259–290

LOCAL LOGICS FOR TRACES [1, 2]

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ABSTRACT

The μ-calculus over dependence graph representation of traces is considered. It is shown that the plain μ-calculus cannot express all monadic second-order (MSO) properties of dependence graphs. Several extensions of the μ-calculus are presented and it is proved that these extensions are equivalent in expressive power to MSO logic. The satisfiability problem for these extensions is PSPACE-complete.

Keywords: Logics for concurrency, traces, mu-calculus

1. Introduction

Infinite words, which are linear orders on *events*, are often used to model executions of systems. Infinite *traces*, which are partial orders on events, are often used to model concurrent systems when we do not want to put some arbitrary ordering on actions occurring concurrently. A *state* of a system in the linear model is just a prefix of an infinite word; it represents the actions that have already happened. A state of a system in the trace model is a *configuration*, i. e., a finite downwards closed set of events that already happened.

Temporal logics over traces come in two sorts: a *local* and a *global* one. The truth of a formula in a *local logic* is evaluated in an event, the truth of a formula in a *global logic* is evaluated in a configuration. Global logics (as for example in [15, 4]) have the advantage of talking directly about configurations hence potentially it is easier to write specifications in them. The disadvantage of global logics is the high complexity of the satisfiability problem [17]. Here we are interested in local temporal logics.

In this paper we present several local logics for traces and show that they have two desirable properties. First, the satisfiability problem for them is PSPACE-complete. Next, these logics are able to express exactly the same properties as monadic second-order logic (MSOL) can.

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[1] Full version of a submission presented at the workshop *Logic and Algebra for Concurrency* held at Dresden University of Technology (Germany), September 13 – 16, 2000.

[2] Big part of this work was done when the author was visiting BRICS, Centre of the Danish National Research Foundation.

[3] The author was supported by Polish KBN grant No. 7 T11C 027 20.