

# THE PARALLEL COMPLEXITY OF DETERMINISTIC AND PROBABILISTIC AUTOMATA <sup>1</sup>

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## ABSTRACT

A deterministic (probabilistic) automaton is said to be in  $TC^0$  whenever its transitions (stochastic event) can be computed by threshold circuits of polynomial size and constant depth. Here, we prove that:

- The class of deterministic automata in  $TC^0$  is closed under homomorphism, sub-automaton, and  $\alpha_0$ -product operations.
- The class of  $k$ -state deterministic (probabilistic) automata is contained in  $TC^0$  if and only if  $k \leq 4$  ( $k \leq 2$ ), unless  $TC^0 = NC^1$ .

Moreover, the possibility of ranking regular languages in  $TC^0$  is related to the group-structure of their syntactic monoid.

*Keywords:* Threshold circuits, deterministic and probabilistic automata, algebraic theory of automata

## 1. Introduction

In this work, we study *the parallel complexity of computing transitions in deterministic automata and stochastic events defined by probabilistic automata*. As computational model, we use *threshold circuits* [18]. We are interested in solving problems by means of threshold circuits with *constant depth*. To this regard, we focus on the class  $TC^0$  [8]

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