TIGHT BOUNDS ON THE SIMULATION
OF UNARY PROBABILISTIC AUTOMATA
BY DETERMINISTIC AUTOMATA [1,2]

Massimiliano Milani

Laboratoire d'Informatique Algorithmique: Fondements et Applications,
Université Paris 7 – Denis Diderot
2, Place Jussieu F-75251 Paris Cedex 05, France
e-mail: `milani@liafa.jussieu.fr`

and

Giovanni Pighizzini

Dipartimento di Scienze dell'Informazione, Università degli Studi di Milano
via Comelico 39, I-20135 Milano, Italy
e-mail: `pighizzini@dsi.unimi.it`

ABSTRACT

In 1963, Rabin proved that any n-state probabilistic automaton, accepting a language with a δ-isolated cut-point, can be simulated by a deterministic automaton with no more than $(1 + 1/\delta)^{n-1}$ states. In this paper, we improve this result in the unary case, namely in the case of automata and languages defined over a one letter input alphabet. We show that any unary n-state probabilistic automaton with an isolated cut-point can be simulated by a deterministic automaton with $O(e^{\sqrt{n \ln n}})$ states in the cyclic part. Furthermore, we show that this result is tight.

We also study the number of states of the initial path of the minimum deterministic automaton which simulates the given n-state probabilistic automaton. We show that this number cannot be bounded by any function of the only parameter n.

Keywords: unary automata, probabilistic automata, descriptional complexity, state complexity.

1. Introduction

Probabilistic models were introduced at the very beginning of information and computer science. In particular, these models are the basis of information theory (see,

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