Journal of Automata, Languages and Combinatorics **6** (2001) 4, 437–452

THE BOOLEAN STRUCTURE OF DOT-DEPTH ONE [1]

CHRISTIAN GLASSER[2]
Theoretische Informatik, Universität Würzburg
Am Hubland, D-97074 Würzburg, Germany
e-mail: glasser@informatik.uni-wuerzburg.de

and

HEINZ SCHMITZ[3]
sd&m AG, software design & management
Thomas-Dehler-Str. 27, D-81737 München, Germany
e-mail: heinz.schmitz@sdm.de

ABSTRACT

By definition, the class $\mathcal{B}_1$ of dot-depth one languages is the Boolean closure of the class $\mathcal{B}_{1/2}$ of languages that can be written as finite unions of $u_0A^+u_1 \dots A^+u_n$, where $u_i \in A^*$. So dot-depth one languages can be described by Boolean combinations of patterns $(u_0, u_1, \dots, u_n)$ in words which captures locally testable and piecewise testable properties. From a descriptional complexity point of view, the lengths of the u_i reflect sequential aspects, while the Boolean operations measure combinatorial complexity.

We prove that the Boolean hierarchy over $\mathcal{B}_{1/2}$ is decidable and strict, which has consequences in first-order logic and complexity theory. Moreover, we effectively characterize the fine structure of $\mathcal{B}_1$ w.r.t. the mentioned sequential and combinatorial measures. This allows the exact location of a given language in this two-dimensional landscape in a computable way.

Keywords: dot-depth hierarchy, Boolean hierarchy, decidability.

1. Introduction

We study starfree regular languages and further investigate one of its subclasses. Let A be some finite alphabet with $|A| \geq 2$. The class $\mathcal{B}_1$ of dot-depth one languages is the Boolean closure of the class $\mathcal{B}_{1/2}$ of languages of A^+ (the set of nonempty words over A) that can be written as finite unions of languages $u_0A^+u_1 \dots A^+u_n$, where $u_i \in A^*$ (the set of words over A) and $n \geq 0$. Dot-depth one is of interest in many fields of research and we give a brief survey.

[1] Full version of a submission presented at the Second International Workshop on *Descriptional Complexity of Automata, Grammars and Related Structures* held in London, Ontario, Canada, July 27 – 29, 2000.

[2] Supported by the Studienstiftung des Deutschen Volkes.

[3] Partially supported by the Deutsche Forschungsgemeinschaft, grant Wa 847/4-1.