CONFLUENT MONADIC STRING-REWRITING SYSTEMS AND AUTOMATIC STRUCTURES

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ABSTRACT

If a monoid M is presented through a finite, special, and confluent string-rewriting system, then the set of irreducible strings is part of an automatic structure for M that is simultaneously $\ell\ell$-, ℓr-, $r\ell$-, and rr-automatic. However, if the presentation is regular instead of finite or monadic instead of special, then the resulting automatic structure is in general only $\ell\ell$- and rr-automatic.

Keywords: automatic monoid, string-rewriting system, confluence.

1. Introduction

We are interested in finitely presented monoids with solvable word problem. As the word problem of a monoid that is given through a finite presentation of the form $(\Sigma; S)$, where Σ is a set of *generators* and S is a set of *defining relations*, is undecidable in general, additional restrictions on the presentations considered are required to guarantee the solvability of the word problem.

Here we will consider two combinatorial restrictions of this kind that have received much attention in the literature and their interrelationship. On the one hand presentations are of interest where the set of defining relations is a string-rewriting system that is *terminating* and *confluent*. If a presentation $(\Sigma; S)$ satisfies this restriction, then the set of irreducible strings $\mathrm{IRR}(S)$ contains a single word from each congruence class mod S, that is, it is a set of unique representatives for the monoid M presented

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