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SOME TOPOLOGICAL PROPERTIES OF RATIONAL SETS

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ABSTRACT

In this paper we give some topological properties of rational sets. The profinite topology was first introduced for the free group by M. Hall, Jr. and by Reutenauer for the free monoid. It is the initial topology for the monoid morphisms from the free monoid into a finite discrete group. For a variety of finite groups **V**, the pro-**V** topology is defined in the same way by replacing "group" by "group in **V**" in the definition. We prove in this paper that the set of accumulation points for the pro-**V** topology of a rational language of the free monoid is a rational language too (if **V** is extension closed) and we give an algorithm to compute it for the profinite and the pro-p topologies. We also give a polynomial time algorithm to test whether a rational set of the free group is the profinite closure of a rational language of the free monoid. Finally we prove that there exist languages which are closed and open and which are not group languages.

Keywords: finite automata, rational languages, formal language theory, profinite topology.

1. Preliminaries

The profinite topology is used to characterize certain classes of rational languages: the languages of level 1/2 in the group hierarchy [22, 24] and the languages recognizable by reversible automata [19]. Moreover pro-**V** topologies play a crucial role in the theory of finite semigroups [1, 10, 14, 20, 29, 30]. In particular, several important decidability problems, related to the Malcev product, reduce to the computation of the closure of a rational language in the profinite topology.

This paper provides a study of some topological aspects of rational languages.

Accumulation points of the Thue-Morse sequence were studied in [7] for the pro-p topology. In the second section of this paper we prove that the set of accumulation points of a rational language is rational too, if we consider the pro-**V** topology, with **V** an extension closed variety of finite groups. Our proof is constructive and it yields a polynomial time algorithm to compute the derived set of a rational language given by a deterministic finite state automaton, for the pro-p or the profinite topology. As another consequence we prove that a sequence of words describing a rational set and which is Cauchy is ultimately constant.