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ON MAXIMAL DENSE BIFIX CODES

YANNICK GUESNET
LIFAR, Université de Rouen
Place Émile Blondel, F-76821 Mont-Saint-Aignan, France
e-mail: yannick.guesnet@dir.univ-rouen.fr

ABSTRACT

A code X is dense if any word $w \in A^*$ is a factor of a word in X, otherwise X is thin. It is known that thin bifix codes cannot be embedded in a maximal one. We prove that any bifix code may be embedded in a maximal dense bifix code, not maximal as code. Our proof lays upon an explicit construction.

Keywords: maximal codes, dense codes, bifix codes, prefix codes.

1. Introduction

In the theory of codes the questions connected to maximality play a prominent part. A code Y is maximal if it is not strictly included in an another code, that is, for any word $y \in A^* \setminus Y$, $Y \cup \{y\}$ does not remain a code. As discussed in the famous book of J. BERSTEL and D. PERRIN "*Theory of Codes*" [2], since any code is included in a maximal one, the structure of these codes is of a fundamental interest. As soon as we deal with maximal codes we are led to partition the family of codes in the two sub-families of thin codes and dense codes. A code X is dense if any word $w \in A^*$ is a factor of a word in X, otherwise X is thin.

From this point of view, a powerful result states that, for any thin codes, being maximal is equivalent to being complete [11]. This result can be extended to some other well-known classes of codes, like thin prefix codes, thin bifix codes [12], thin circular codes [7], thin codes with finite deciphering delay [3] and thin uniformly synchronous codes [4].

A classical problem on maximal codes consists in the embedding of a code in a maximal one. More precisely, let $\mathcal{F}$ be a family of code. Given a code $X \in \mathcal{F}$, the problem is to construct a code $Y \in \mathcal{F}$ containing X such that Y is maximal in $\mathcal{F}$. In the case where $\mathcal{F}$ is one of the preceding families, various constructions are known [13, 1, 4, 6, 8, 10]. However, in all of these constructions, $\mathcal{F}$ is included in the family of thin codes. Notice that since, in this families, a code is maximal iff it is complete, these embedding methods come down to the construction of a complete code. Recently we have presented such a method for the so-called class of thin codes with finite interpreting delay [9, 10].