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THE DEPTH OF A HYPERSUBSTITUTION

Klaus Denecke, Jörg Koppitz
Fachbereich Mathematik, Universität Potsdam
Postfach 60 15 53, D-14415 Potsdam, Germany
e-mail: {kdenecke, koppitz}@rz.uni-potsdam.de

and

Slavcho Shtrakov
Faculty of Mathematics and Natural Sciences, South-West-University Blagoevgrad
BG-2700 Blagoevgrad, Bulgaria
e-mail: shtrakov@aix.swu.bg

ABSTRACT

For given depths of the terms $s, t_1, \ldots, t_n$ a formula will be proved to calculate the depth of the composed term $s(t_1, \ldots, t_n)$ and if σ is a hypersubstitution and t is a term we derive a formula for the depth of $\hat{\sigma}[t]$.

Keywords: depth of a term, high of a tree, composition of terms, hypersubstitution.

1. Introduction

At first we remember of the following definition of terms. Let $X = \{x_1, \ldots, x_n, \ldots\}$ be any countably infinite (standard) alphabet of variables and let $X_n = \{x_1, \ldots, x_n\}$ be an n-element alphabet. Let $(f_i)_{i \in I}$ be an indexed set which is disjoint from X. Each f_i is called an n_i-ary operation symbol where $n_i \geq 1$ is a natural number. Let τ be a function which assigns to every f_i the number n_i as its arity. The function τ or the sequence of values of τ, written as $(n_i)_{i \in I}$, is called a type. An n-ary term of type τ is defined inductively as follows:

(i) The variables $x_1, \ldots, x_n$ are n-ary terms.

(ii) If $t_1, \ldots, t_m$ are n-ary terms and if f_i is an n_i-ary operation symbol then $f_i(t_1, \ldots, t_{n_i})$ is an n-ary term.

(iii) Let $W_\tau(X_n)$ be the smallest set which contains $x_1, \ldots, x_n$ and is closed under finite application of (ii). Every $t \in W_\tau(X_n)$ is called an n-ary term of type τ.

We remark that by this definition every n-ary term is also $(n+1)$-ary. The set $W_\tau(X) := \bigcup_{n=1}^{\infty} W_\tau(X_n)$ is the set of all terms of type τ.