# PIECEWISE DIRECTABLE AUTOMATA

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ABSTRACT

In this paper a new strong form of directability of automata is studied. A word $w$ is *piecewise directing* if any input word containing $w$ as a piecewise subword takes the automaton to the same state from every state. The *piecewise directable automata* form a variety of automata situated between the varieties of definite automata and directable automata. We describe the sets of piecewise directing words of automata in terms of Haines' embedding order, study the structure of piecewise directable automata, and present criteria for an automaton to be piecewise directable. We also consider the congruences of an automaton which yield a piecewise directable quotient automaton and show that any $n$-state piecewise directable automaton has a piecewise directing word of length $\leq \binom{n}{2}$.

*Keywords:* directable automata, piecewise directing words, regular languages.

## 1. Introduction

An automaton is said to be *directable* (or *synchronizable*) if it has a word, called a *directing word*, which takes the automaton from every state to the same state. In the terminology of Moore experiments [12] a directing word is a special homing sequence which yields a known final state even if no outputs are observed. It seems that the notion of directability has occurred independently at different times to many authors. Some of the earliest sources are [11] and [5]. In [5] ČERNÝ states his famous conjecture claiming that any $n$-state ($n \geq 1$) directable automaton has a directing word of length $\leq (n-1)^2$. Although $(n-1)^2$ is shown to be a lower bound already in [5], the best upper bounds known for the length of the shortest directing words are still of order $n^3$. On the other hand, for some special classes considerably better, and even exact, bounds are known (cf. [13, 15, 9] and [8], for example). For a general survey of the theory of directable automata and many references the reader is referred to [2].

In this paper we introduce and study a stronger form of directability. A word $w$ is said to be *piecewise directing* (pwd) for an automaton $\mathcal{A}$ if any input word which contains $w$ as a piecewise subword is a directing word of $\mathcal{A}$, that is to say, if the input

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