Journal of Automata, Languages and Combinatorics **6** (2001) 1, 97–114

FUNCTION-LIMITED 0L SYSTEMS REVISITED

DIETMAR WÄTJEN and DAVID OSTROVSKY
Institut für Theoretische Informatik, Technische Universität Braunschweig
Postfach 33 29, D-38023 Braunschweig, Germany
e-mail: waetjen@iti.cs.tu-bs.de

ABSTRACT

Function-limited and uniformly function-limited 0L systems have been introduced by FERNAU and STIER, respectively. In this paper, these systems are revisited concentrating on language families generated by such systems with fixed limitation functions. For different limitation functions, the corresponding language families are compared with each other as well as with other language families. Closure properties are investigated. Moreover, the corresponding extended systems are considered. Some results also solve problems left open by FERNAU and STIER.

Keywords: formal languages, function-limited 0L systems.

1. Introduction

Function-limited Lindenmayer systems have been introduced by FERNAU [1] as a generalization of the k-limited 0L systems of WÄTJEN [9]. In a similar manner, STIER [8] considered uniformly function-limited Lindenmayer systems as a generalization of uniformly k-limited 0L systems [10]. All these systems impose a limitation upon the parallel rewriting of normal 0L systems. In the case of function-limited systems, the limitation is separately imposed upon any symbol of the alphabet of the system occurring in the word considered while in the case of uniformly function-limited systems, we do not distinguish between the different symbols. FERNAU and STIER do not investigate families of languages generated by systems with one fixed limitation function, but instead they focus on language families generated by 0L systems with certain classes of limitation functions. FERNAU has shown (see [1], Theorem 2.1) that there exist total non-computable functions such that the generated languages are not recursively enumerable. Instead, if the limitation function is total and computable, then the generated language always is recursively enumerable ([1], Theorem 2.2). This fact has lead FERNAU and STIER to the restrictions imposed upon the considered limitation functions. As the biggest class they consider

$$\mathscr{N} = \{f : \mathbb{N} \to \mathbb{N} \mid f \text{ is total and computable}\}.$$

(To state it more accurately, they use the functions $f : \mathbb{N} \to \mathbb{N}_0$, see Section 2.) For a class $\mathscr{F}$ of functions $f : \mathbb{N} \to \mathbb{N}$, they set

$$\mathscr{F}\mathscr{B} = \{f : \mathbb{N} \to \mathbb{N} \mid f \text{ total and computable},\ \exists g \in \mathscr{F}\ \forall n \in \mathbb{N} : f(n) \leq g(n)\}.$$