Journal of Automata, Languages and Combinatorics **6** (2001) 1, 3–23


# REAL-TIME AUTOMATA [1]

CĂTĂLIN DIMA
*Department of Fundamentals of Computer Science, Bucharest University*
*Str. Academiei 14, R-70109 Bucharest, Romania, and*
*INRIA Rhône-Alpes, 655 Av. de l'Europe, F-38330 Montbonnot St. Martin, France*
*e-mail:* cdima@funinf.math.unibuc.ro

ABSTRACT

We study here the class of timed automata with a single clock which is reset at each transition. We adapt for these automata the classical results for finite automata: the Kleene Theorem, the closure under complementation and the Pumping Lemma. We provide an algorithm for the elimination of stuttering steps, which is essential in complementation. This algorithm relies upon the properties of the Kleene algebra of sets of real numbers, namely the existence of a normal form for sets of reals generated from intervals with rational bounds, using boolean operations, summation and star.

*Keywords:* real-time automata, rational expressions, Kleene algebra.

## 1. Introduction

The search for an appropriate class that may bear the name of *real-time regular languages* has produced several results like [1, 2, 15]. The features sought for were: closure under complementation and/or relationship to some monadic logic of order over the real-time axis that generalizes MSO over words [20, 21]. The relationship with some rational expressions was not under attention when assigning the label *regular* to a class of languages. It is already known [3, 5, 11] that rational (or regular) expressions do not easily fit the classes of timed automata or event-clock automata. This amounts to the problem of giving semantics for concatenation: in the presence of a total operation like in [3] one direction of the Kleene Theorem is missing [16]. For obtaining both directions of the Kleene Theorem a *partial* operation on timed words or signals is needed for the semantics of concatenation, see [5, 11].

In this paper we study a class of automata which we claim to be the largest extension from finite automata still carrying the decidability of both the emptiness and the universality problems, a Pumping Lemma and, moreover, a Kleene Theorem in which the semantics of the associated rational expressions is based upon a total "concatenation" operation. The automata we study, called Real-Time Automata (RTA), can be regarded as timed automata with a single clock which is reset at each transition, and

[1] A preliminary version of this paper appeared as [12].