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# LOCALLY MAXIMAL CLONES II

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ABSTRACT

For a general local completeness criterion for algebras on an infinite universe $A$ we need to know all locally maximal clones on $A$ as well the increasing chains of proper local clones whose union is locally complete. The clones in question are all of the form $\mathrm{Pol}\rho$ where $\rho$ is a finitary relation on $A$ and $\mathrm{Pol}\rho$ is the set of all operations on $A$ preserving $\rho$. The predecessor paper Local completeness I left the following 5 sets of relations on $A$ to be sorted out: the locally bounded graphs, diagraphs and reflexive digraphs of diameter 2, a set of ternary relations and the pivotal set of all totally reflexive and symmetric relations. We present partial results for each of the 5 types. For graphs they relate to largest infinite cliques and digraphs are restricted to the acyclic ones which either have arctransitive endomorphisms or a certain vanishing-interval property. We restrict the last type to two kinds of relations termed strongly homogeneous and protective and we find many instances of the increasing chains of proper local clones mentioned earlier.

*Keywords:* local completeness, locally maximal clone, preservation of relations.

## 1. Introduction and Preliminaries

The paper is devoted to the search for a general local completeness criterion for universal algebras on a given infinite universe $A$. Such a criterion is provided by a cofinal subset of the poset of local clones distinct from the greatest clone $\mathcal{O}_A$ of all finitary operations on $A$. A "small" cofinal set consists of all locally maximal (also called precomplete or preprimal) clones as well as certain towers, (increasing chains of local clones whose union is locally complete).

The paper starts off where [9] ended – namely with 5 types of relations on $A$ listed below – and proceeds by an elimination based on relational constructions. A pivotal part is the elimination in the set $T$ of totally reflexive and totally symmetric relations on $A$ which is likely to produce most of local clones and towers. In Section 7 a variant