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ON DEHN FUNCTIONS OF FINITELY PRESENTED BI-AUTOMATIC MONOIDS

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ABSTRACT

For each automatic monoid the word problem can be solved in quadratic time (Campbell et al. 1997), and hence, the Dehn function of a finitely presented automatic monoid is recursive. Here we show that this result on the Dehn function cannot be improved in general by presenting finitely presented bi-automatic monoids the Dehn functions of which realize arbitrary complexity classes that are sufficiently rich.

Keywords: automatic monoid, string-rewriting system, derivation, Dehn function, recursive function, sufficiently rich complexity class.

1. Introduction

It is well-known that in general it is not possible to extract much information on the algebraic structure of a monoid or a group from a given finite presentation for that monoid or group. On the other hand various methods have been developed that can be applied successfully to certain restricted instances.

One of the classical methods is the *Todd-Coxeter method* of enumerating the cosets of a group G with respect to a finitely generated subgroup H. In case that H has finite index in G this method yields a complete coset table, which in particular gives complete information on how the group elements of G act on the cosets of H in G. If H is taken to be the trivial subgroup, then this method yields the multiplication table of G, if G is a finite group.

During the 1980's Epstein et al. developed the notion of a *group with an automatic structure* [6], which can be seen as a generalization of the Todd-Coxeter coset enumeration method. A finitely generated group G is said to have an *automatic structure* if it has a finite set of generators Σ and a regular set $C \subseteq \Sigma^*$ of (not necessarily unique) representatives for the group G such that the following tasks can be performed by finite state acceptors (fsa's):

(1) Given two elements $u, v \in C$, decide whether or not they both define the same element of the group G.

(2) For each generator $a \in \Sigma$, if two elements $u, v \in C$ are given, decide whether or not $u \cdot a$ and v represent the same element of the group G.