RESTRICTED TEMPORAL LOGIC
AND DETERMINISTIC LANGUAGES [1]

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ABSTRACT

In formulas of restricted temporal logic (RTL) arbitrary use of the temporal operators *next* (**X**), *eventually* (**F**) and Boolean connectives is allowed. We show in detail to what extent the *next* operator contributes to the expressive power of this logic and reveal close connections to formal languages and finite automata.

Let $\mathrm{TL}[\mathbf{X}(k), \mathbf{F}]$ denote the class of formulas with nesting depth at most k in **X**. For all $k \geq 0$ we characterize the class of $\mathrm{TL}[\mathbf{X}(k), \mathbf{F}]$-definable languages (1) in terms of a formal language representation involving a generalization of the notion of deterministic languages, and (2) in terms of a certain pattern that must not appear in the transition graph of deterministic finite automata. This leads to concise proofs of strictness and decidability results for this *next* hierarchy, which exhausts the class of RTL-definable languages. We discuss relations to languages having dot-depth one.

Keywords: Temporal logic, finite automata, deterministic languages, forbidden pattern.

1. Introduction

The control of events occurring over time and the formalization of this process is a fundamental task in computer science. We contribute to the study of temporal logic and investigate the ability of one of its fragments to express temporal properties. This can be seen in connection with recently surveyed results [21], see also [20], where the expressive power of fragments obtained by omitting one or the other of the usual temporal operators *next* (**X**), *eventually* (**F**) and *until* (**U**) have been studied. In particular, several proofs are known for the fact that formulas involving all three operators together with Boolean connectives (interpreted over finite words) yield the starfree regular languages, see [9, 11, 8, 5, 21]. Restricting the nesting depth in **U** was treated in [7, 19]. Moreover, characterizations are known for the two cases when **U** is omitted [5], so-called restricted temporal logic (RTL), and when both, **U** and **X** may not be used [5, 7]. In the latter case we are not allowed to specify the *next*