

FORMAL LANGUAGES OVER FREE BINOIDS¹

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ABSTRACT

We introduce new algebraic systems named bisemigroups, bimonoids and binoids. A bisemigroup consists of a set of objects and two associative operations. A bimonoid is a bisemigroup which has an identity to each operation. A binoid is a bimonoid in which the two identities coincide. In accordance with the well known hierarchy of formal grammars over free monoids, we shall introduce five types of grammars for defining formal languages over free binoids, and present many basic properties of these languages. Especially we present several relations between these languages and Turing machines and finite automata.

Keywords: Formal language, bisemigroup, binoid, Turing machine, finite automaton.

1. Introduction

Formal language theory consists of studies of formal grammars generating formal languages (which are subsets of free monoids) and automata accepting formal languages. The grammars constitute the so-called Chomsky hierarchy. The theory has been developed very widely and deeply, and contributed to many applications in computer science. In this work, we shall study new types of languages which are subsets of free binoids. In fact, we shall introduce three new algebraic systems named bisemigroups, bimonoids and binoids. A bisemigroup is an algebraic system which has two associative operations. A bimonoid is a bisemigroup which an identity to each associative operation. A binoid is a bimonoid in which the two identities coincide. The motivation of this study is to find new possibilities for improving descriptive systems [2, 3, 4, 6]: we are also interested to study new types of algebraic systems, bisemigroups, bimonoids and binoids [8]. We shall call a subset of any free binoid a binoid language, or very often a B-language. Sometimes we shall use the words “monoid language” for any subset of any free monoid.

According to the Chomsky hierarchy, we shall introduce phrase structure B-grammars, context-sensitive B-grammars, context-free B-grammars, right linear B-grammars and left linear B-grammars; however, the definitions are not completely analo-

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