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SOME PROPERTIES OF LEFT NON-CANCELLATIVE LANGUAGES [1]

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ABSTRACT

Let X^* be the free monoid generated by an alphabet X consisting of more than one letter. Let $X^+ = X^* \setminus \{1\}$, where 1 is the empty word. The family of languages $\mathcal{M} = \{L \mid L \subseteq X^+ \text{ or } L = \{1\}\}$, is a monoid under the catanation operation, which is known as the monoid of languages. A language $L \in \mathcal{M}$ is left cancellative if $LL_1 = LL_2$ for $L_1, L_2 \in \mathcal{M}$ always implies that $L_1 = L_2$. SHYR has shown that for a language L which is left non-cancellative always exists a word $u \in X^+$ such that $LX^+ = LX_u^+$, where $X_u^+ = X^+ \setminus \{u\}$. This paper is a study of some algebraic properties of left non-cancellative languages. We define for a given word $u \in X^+$, the set $\mathcal{L}_u = \{L \in \mathcal{M} \mid LX^+ = LX_u^+\}$ and for a left non-cancellative language $L \in \mathcal{M}$, the set $\mathrm{Nul}(L) = \{u \in X^+ \mid LX^+ = LX_u^+\}$. We show that $\mathcal{L}_u$ is a left ideal of $\mathcal{M}$ and $\mathrm{Nul}(L)$ is a right ideal of X^*. We give a characterization of a language L which is in $\mathcal{L}_u$ and a characterization of a word u which is in $\mathrm{Nul}(L)$. A characterization of maximal null languages is also obtained. Two languages L_1 and L_2 in $\mathcal{M}$ are said to be null-equivalent if $\mathrm{Nul}(L_1) = \mathrm{Nul}(L_2)$. The null-equivalent relation is an equivalence relation defined on $\mathcal{M}$ which is not a left congruence relation and also not a right congruence relation. A characterlization of two languages being null-equivalent is obtained.

Keywords: Monoid of languages, left cancellative language, equivalence relation, regular.

1. Introduction

Let X be an alphabet consisting of more than one letter and let X^* be the free monoid generated by X. Let $X^+ = X^* \setminus \{1\}$, where 1 is the empty word. Any subset of X^* is a *language* and for any two languages L_1 and L_2, the *catenation* of L_1 and L_2 is the set $L_1L_2 = \{xy \mid x \in L_1,\ y \in L_2\}$. We are interested in the family of languages $\mathcal{M} = \{L \mid L \subseteq X^+ \text{ or } L = \{1\}\}$, which is a monoid under the catenation operation and call it the *monoid of languages* (see [3]). A language $L \in \mathcal{M}$ is a *prefix code* if $L \cap LX^+ = \emptyset$. Following SHYR, we call a language $L \in \mathcal{M}$ *left cancellative* if $LL_1 = LL_2$, $L_1, L_2 \in \mathcal{M}$, always implies that $L_1 = L_2$. The family of left cancellative

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