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# COVERING SUBMONOIDS AND COVERING CODES

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ABSTRACT

This paper deals with the formalization of the intuitive notion of covering monoid and the investigation of the related algebraic properties. It is shown that covering monoids can be regarded as a generalization of the well known classical monoids and z-monoids. A new coding notion is introduced and a simple method to decide whether a finite set $X$ of words is a covering code is described. Finally, the case in which $X$ is an uniform set, i.e. a set whose elements are all of the same length, is analyzed.

*Keywords:* Codes, string covering, covering codes.

## 1. Introduction

The aim of this paper is to formalize the intuitive concept of covering. From an elementary point of view, given a word $w$ on an alphabet $A$, the question is whenever $w$ can be "covered" by words taken from a given set $X$, i.e. whether it is possible to obtain $w$ overlapping the elements of a suitable sequence $x_1, \ldots, x_n$ of words of $X$ (see [7, 2]). Informally speaking, a covering monoid $X^{\mathrm{cov}}$ is obtained by overlapping in all possible ways words of $X$.

Starting from this primitive idea, it is quite natural that the introduction of rules in performing the overlaps gives rise to some structures that can be characterized from an algebraic point of view. For example, if we impose that all overlaps must be empty, covering monoids coincide with classical monoids with respect to the well-known operation of concatenation.

In this paper we develop the case in which no rules are imposed in the choice of the overlaps, as in the more extensive concept of covering. Taking into account this more general case, we can consider covering monoids as a generalization of well known classical ones (see [3, 6]) and z-monoids (see [1, 9, 8, 4, 10, 11, 12]). This is the reason why we have chosen to approach our algebraic investigation starting from