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ON LEFTIST SIMPLY GENERATED TREES

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ABSTRACT

We shall derive a functional equation for the bivariate (resp. univariate) generating function associated with the number of leftist trees having a specified number of leaves and nodes (resp. of leaves) appearing in an arbitrary family of simply generated trees. In the case that only the leaves are specified we shall present an asymptotical equivalent to the corresponding number of leftist trees. That general result is illustrated by computing explicit values for the exact and asymptotical number of leftist trees belonging to various families of simply generated trees.

Keywords: Leftist trees, simply generated trees, functional equation, asymptotical analysis.

1. Introduction and Basic Definitions

A lot of classes of unlabelled rooted ordered trees playing a part in Computer Science can be interpreted as a representation of a special simply generated family of trees introduced in [12]. A family $\mathcal{F}$ of rooted trees is said to be *simply generated* if the generating function $E(z) := \sum_{n\geq 1} t(n)z^n$ of the number $t(n)$ of all trees $\tau \in \mathcal{F}$ with n nodes satisfies a functional equation of the form $E(z) = z\,\Theta(E(z))$, where $\Theta(y) := \sum_{\lambda\geq 0} c_\lambda\, y^\lambda$ is a regular function when $|y| < R < \infty$ with $c_0 = 1$, $c_\lambda \geq 0$ for $\lambda \in \mathbb{N}$, and $c_\lambda > 0$ for some $\lambda \in \mathbb{N}\setminus\{1\}$. This definition obviously includes the most common classes of trees such as *t-ary trees* ($\Theta(y) := 1 + y^t$, $t \in \mathbb{N}\setminus\{1\}$), *extended binary trees* ($\Theta(y) := 1 + y^2$), *binary trees* ($\Theta(y) := (1+y)^2$), *unary-binary trees* ($\Theta(y) := 1 + y + y^2$), *unbalanced 2-3-trees* ($\Theta(y) := 1 + y^2 + y^3$) and *ordered trees* ($\Theta(y) := (1-y)^{-1}$). Given the regular function Θ, the corresponding simply generated family of trees $\mathcal{F}(\Theta)$ is completely characterized: the elements of the set $\mathcal{DEG}(\Theta) := \{\lambda \in \mathbb{N}_0 \mid \langle y^\lambda; \Theta(y)\rangle \neq 0\}$[1] are the allowed node degrees in the trees appearing in $\mathcal{F}(\Theta)$ and $c_\lambda = \langle y^\lambda; \Theta(y)\rangle$, $\lambda \in \mathbb{N}_0$, reflects whether different orderings of the λ edges of a node of degree λ are taken into account in distinguishing between the trees in $\mathcal{F}(\Theta)$. Here, the *degree* $\deg(x)$ of a node x is the number of its sons. A node is called an *internal node* (resp. *leaf*) if $\deg(x) > 0$ (resp. $\deg(x) = 0$). Using the

[1] The abbreviation $\langle z_1^{n_1}\dots z_m^{n_m}; f(z_1,\dots,z_m)\rangle$ denotes the coefficient of $z_1^{n_1}\dots z_m^{n_m}$ in the expansion of $f(z_1,\dots,z_m)$ at $(z_1,\dots z_m) = (0,\dots,0)$.