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AN AUTOMATA THEORETIC DECIDABILITY PROOF FOR FIRST-ORDER THEORY OF $\langle \mathbb{N}, <, P \rangle$ WITH MORPHIC PREDICATE P [1]

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ABSTRACT

We show connections between morphisms on words and pictures on a finite alphabet and finite deterministic incomplete automata. We use these connections to re-prove, in terms of automata, a decidability result about the first-order theory of the structures $\langle \mathbb{N}, <, P \rangle$ for multi-ary morphic predicates P.

Keywords: Morphism, automaton, word, picture, decidability.

1. Introduction

In a recent paper [14], we proved the following result: given a unary morphic predicate P, the first-order theory of the structure $\langle \mathbb{N}, <, P \rangle$ is decidable. A unary morphic predicate is a unary predicate whose characteristic word is the projection on the alphabet $\{0, 1\}$ of the fixed point of a morphism on words on a finite alphabet.

In order to prove this result, we introduced a notion of morphism on pictures (that is, multi-dimensional words), as well as a notion of *shape-symmetry* for morphic pictures. This notion of symmetry is related to the growth of the morphism in each direction. The main result of [14] is that, for any multi-dimensional shape-symmetric morphic picture P, the first-order theory of the structure $\langle \mathbb{N}, <, \mathcal{P} \rangle$ is decidable, where $\mathcal{P}$ is the set of all projections of P on the alphabet $\{0, 1\}$. This implies the above statement because any morphic word is a shape-symmetric morphic picture.

Examples of unary morphic predicates include the set of powers of a natural number, the set of FIBONACCI numbers and the set of square numbers. Similarly, the PASCAL triangle modulo 2 (considered as a binary relation) and the ternary relation of addition are other examples of morphic predicates. See [3, 6, 1, 13].

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