

## QUASIPERIODICITY: FROM DETECTION TO NORMAL FORMS<sup>1</sup>

COSTAS S. ILIOPOULOS

*Dept. of Computer Science, King's College London, Strand, London WC2R 2LS, UK*  
*School of Computing, Curtin University, GPO Box U1987, Perth 6845, WA, Australia*  
*e-mail: csi@dcs.kcl.ac.uk*

and

LAURENT MOUCHARD<sup>2</sup>

*LIFAR – ABISS, Faculté des Sciences, Université de Rouen*  
*F-76821 Mont Saint Aignan Cedex, France*  
*Dept. of Computer Science, King's College London, Strand, London WC2R 2LS, UK*  
*e-mail: lm@dir.univ-rouen.fr*

### ABSTRACT

In this paper, we focus on strings regularities and in particular extension of the notion of the periodicity: quasiperiodicity. A string  $w$  is *quasiperiodic* if it can be constructed by concatenation and superpositions of one of its proper factor. We recall the  $O(n \log^2 n)$  algorithm for the detection of quasiperiodicity in strings due to APOSTOLICO and EHRENFUCHT [2] and present our new approach of this problem, which leads to an  $O(n \log n)$  algorithm.

We also present the left and right normal form of quasiperiodic strings, an efficient algorithm for computing these forms, the left and right normal form of covered strings and some applications which are built on these notions.

*Keywords:* Regularities, periodicity, quasiperiodicity, superposition, normal forms, algorithms.

### 1. Introduction

Early in this century, AXEL THUE considered several combinatorial problems related to repetitions in finite and infinite strings. Part of his work was the study of avoidable and unavoidable regularities in sequences. One can easily see that squares are unavoidable regularities over a binary (2-letter) alphabet  $\mathcal{A} = \{A, T\}$  since the only strings which are square-free are the empty string  $\epsilon$ , A, T, AT, TA, ATA and TAT. But no string of length at least 4 avoids squares.

---

<sup>1</sup>Full version of a submission presented at the “Mons’ Days of Theoretical Computer Science” at the University of Mons-Hainaut, Belgium, March 2–4, 1998.

<sup>2</sup>Partially supported by the Conseil Régional de Haute-Normandie and by Programme Génome (C.N.R.S.).