Journal of Automata, Languages and Combinatorics 4 (1999) 3, 205–211


# SEQUENCES OBTAINED FROM A SERIES OF PERMUTATIONS OF BORDERS AND ULTIMATELY PERIODIC WORDS [1]

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ABSTRACT

A word of ***length*** $n$ over an alphabet $A$ is a sequence $x = a_1 \dots a_n$ of letters of $A$. A "long enough" or one-sided word over $A$ is an infinite right word, that is an infinite sequence $a_1 \dots a_i \dots$ of elements of $A$. An integer $p$ is a ***period*** of the word in the interval $[j \cdot\cdot k]$ if we have $a_i = a_{i+p}$ for those indices $i$ and $i + p$ in the considered interval. An infinite word is ***ultimately periodic*** with period $p$ if for a given integer $j$ the word $a_j \dots$ has period $p$. A word $u$ is a ***border*** of a word $w$ if $u$ is both prefix and suffix of this word, that is $w = u \cdot x = y \cdot u$ for two words $x$ and $y$. The word $w' = x \cdot u$ is obtained from the word $w = u \cdot x = y \cdot u$ by the ***permutation of border*** $u$.

The question of interest here is to know if a sequence constructed from an initial word $w$ by iterating permutation of border is constant from a certain rank. The results exposed here are an unpublished answer we offered to M. P. SCHÜTZENBERGER to a question concerning the characterization of the period of an ultimately periodic word.

*Keywords:* Periodicity, permutation of border, ultimately periodic word.

## 1. Introduction

A word $u$ is a *border* of a word $w$ if and only if $u$ is both prefix and suffix of $w$, that is, there exist two words $x$ and $y$ such that $w = u \cdot x = y \cdot u$. Any word has at least two *trivial* borders: the empty word $\epsilon$ and the word itself, but the borders

[1]Full version of a submission presented at the "Mons' Days of Theoretical Computer Science" at the University of Mons-Hainaut, Belgium, March 2–4, 1998.

[2]Supported by ABISS.

[3]Partially supported by CNRS Programme Informatique et Génome.