

## DECISION ISSUES ON FUNCTIONS REALIZED BY FINITE AUTOMATA <sup>1</sup>

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### ABSTRACT

Consider a numeration system and a finite set of symbols. Each finite (resp. infinite) sequence on this set represents an integer (resp. a real). Synchronous two-tape automata are devices that define a sequence-to-sequence mapping and can thus be interpreted as performing a relation on integers (resp. reals).

Given a numeration system belonging to some natural family defined in this paper and a synchronous two-tape automaton, we show that the following questions are decidable in polynomial time: whether the relation is a function and if this is the case whether it is monotone, injective, continuous (for the reals).

*Keywords:* Numeration systems, on-line functions of numbers, synchronous automata, decision problems.

### 1. Introduction

Let  $D$  be some finite alphabet of symbols, (a set of “digits”). A numeration system is a function  $\pi$  that associates with each sequence of symbols of  $D$ , a number of which it is a representation. Consider further a mapping  $f$  from sequences to sequences performed by some device. We say this device computes a numerical function or (in

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