Journal of Automata, Languages and Combinatorics 4 (1999) 2, 111–116

HAJÓS FACTORIZATIONS OF CYCLIC GROUPS – A SIMPLER PROOF OF A CHARACTERIZATION

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ABSTRACT

A pair (T, R) of subsets of a finite abelian group G is a factorization of G if G is the direct sum of T and R. A class of factorizations of finite cyclic groups were described by HAJÓS and a new characterization of these factorizations arose in the framework of the theory of variable-length codes. In this note we will give a simpler proof of this characterization.

Keywords: variable-length codes, finite maximal codes, factorizations of finite cyclic groups.

1. Introduction

In this note we consider the problem of finding a description of the pairs (T, R) of subsets of a finite cyclic group $\mathbb{Z}_n$ such that for any $z \in \mathbb{Z}_n$ a unique pair (t, r) exists with $t \in T$, $r \in R$ and $t + r = z$. Any such pair is called a *factorization* of $\mathbb{Z}_n$ [7]. This question was raised by HAJÓS along with a method for constructing a class of factorizations [8, 13]. Subsequently, it has been proved that the HAJÓS construction does not produce all the factorizations of $\mathbb{Z}_n$ (see [7] for further details).

Describing the structure of the subsets T, R in a factorization remains an open problem and it is still of great interest in the field of the theory of groups (see [3, 4] for recent results). In addition, this problem is also intriguing for the theoretical computer scientist. Indeed, a relationship exists between factorizations of cyclic groups and *variable-length codes.*

In the algebraic framework initiated by SCHÜTZENBERGER, a code C is the base of a free submonoid of a free monoid A^* ([14], see [1] for a systematic exposition of this theory and [2] for a recent survey on this topic). A still open and important conjecture, the *factorization conjecture*, proposed by SCHÜTZENBERGER, states the equality between finite *maximal* codes and the so-called *factorizing codes* [11, 12]. As an example of the above-mentioned relation between codes and factorizations, the

[1]Partially supported by Project 60% MURST and by Project "Codage et concurrence / Codici e Concorrenza" CGRI – MAE, MURST.