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PARIKH'S THEOREM DOES NOT HOLD FOR MULTIPLICITIES

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ABSTRACT

We consider the question of whether the famous PARIKH's Theorem holds with multiplicities i.e., for formal power series instead of languages. The strict hierarchy of algebraic, rational, recognizable and semilinear formal power series in commuting variables is proved and in this way it is established that the PARIKH's Theorem does not hold with multiplicities. We also characterize the recognizable series over a product monoid giving a generalization of the MEZEI's Theorem.

Keywords: PARIKH's Theorem, multiplicities, formal power series, MEZEI's Theorem.

1. Introduction

The starting point of this study is the famous PARIKH's Theorem, which states that the commutative images of the context-free languages equal the commutative images of the regular languages, cf. [8]. Moreover, it can be proved that these images are the semilinear sets of vectors with non-negative integers as components. This is a very nice but in a sense lossy result since the family of context-free languages is larger than the family of regular languages.

That is why, in order to catch more details about a language, we consider here also the multiplicity (or the ambiguity) of a word. This means that we will retain not only whether or not a word is accepted (or generated) by a device but also how many times it is accepted (or generated). In this way we consider here instead of languages, formal power series with non-negative integers as coefficients. Since we are interested in the PARIKH image, we will deal with series over a free commutative monoid $\mathbb{M} = \Sigma^{\oplus} = a_1^* \times \cdots \times a_n^*$, with $\Sigma = \{a_i \mid 1 \leq i \leq n\}$ a nonempty alphabet. We prove that the hierarchy of algebraic, rational, recognizable and semilinear formal power series in commuting variables is strict. This means in particular that the PARIKH's Theorem does not hold with multiplicities.

It is worth noticing here that there are in literature some extensions of the PARIKH's Theorem to formal power series. For example, in [4] and [5], it is proved that the PARIKH's Theorem holds in an idempotent, commutative, ω-continuous semiring. Our new idea of considering multiplicities in the classical PARIKH's Theorem will lead us to