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ON THE SYSTEM OF WORD EQUATIONS $x_0u_1^ix_1u_2^ix_2\ldots u_m^ix_m = y_0v_1^iy_1v_2^iy_2\ldots v_n^iy_n$ $(i = 0, 1, 2, \ldots)$ IN A FREE MONOID

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ABSTRACT

We prove that if $x_0, x_1, \ldots, x_m, u_1, u_2, \ldots, u_m, y_0, y_1, \ldots, y_n, v_1, v_2, \ldots, v_n$ are words such that the equality $x_0u_1^ix_1u_2^ix_2\ldots u_m^ix_m = y_0v_1^iy_1v_2^iy_2\ldots v_n^ix_n$ holds for $i = 0, 1, 2, \ldots, m+n+2$, then it is true for each $i = 0, 1, 2, \ldots$

Keywords: word equation, conjugate, primitive root.

0. Introduction

Classical examples of language families whose elements possess some kind of a pumping property are regular, context-free, bounded, and commutative languages. When considering, for instance, the decidability of morphism (or some other mapping) equivalence or effective existence of a test set for those languages, we are led to systems of word equations where pumping in one or several points in an equation can appear.

In this paper we study the general system of word equations

$$x_0u_1^ix_1u_2^ix_2\ldots u_m^ix_m = y_0v_1^iy_1v_2^iy_2\ldots v_n^iy_n \quad (i = 0, 1, 2, \ldots) \qquad (*)$$

where m and n are positive integers, $x_0, x_1, \ldots, x_m, y_0, y_1, \ldots, y_n$ are words over a finite alphabet X (the *midwords* of the system) and $u_1, u_2, \ldots, u_m$, $v_1, v_2, \ldots, v_n$ are nonempty words over X (the *loops* of the system).

By the validity of EHRENFEUCHT Conjecture [2, 10], there exists a finite subsystem of $(*)$ such that if each equation in the subsystem is satisfied, then $(*)$ is also true. Let us have a look what is known about the special cases of the general system $(*)$ of word equations. In [1] it is shown that if $u_1^n = v_1^nv_2^n\ldots v_n^n$, then $u_1^i = v_1^iv_2^i\ldots v_n^i$ for all $i = 1, 2, 3, \ldots$ The results of [5] imply that if $u_1^iu_2^i\ldots u_m^i = v_1^iv_2^i\ldots v_n^i$ for $i = 1, 2, \ldots, \lceil\frac{r}{2}\rceil + 1$, where $r = \max\{m, n\}$, then $u_1^iu_2^i\ldots u_m^i = v_1^iv_2^i\ldots v_n^i$ for all $i = 1, 2, \ldots$ The paper [6] considers $(*)$ when $\max\{m, n\} = 3$. It is proved that if the equality $x_0u_1^ix_1u_2^ix_2u_3^ix_3 = y_0v_1^iy_1v_2^ix_2v_3^iy_3$ holds for $i = 0, 1, 2, 3, 4, 5$, then it is true also for $i = 6, 7, 8, \ldots$