

## ON THE FREQUENCY OF FACTORS IN A D0L WORD <sup>1</sup>

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### ABSTRACT

We study the frequency of factors in a D0L word on an alphabet  $\Sigma$ , which is known to constitute a probability measure on the set of all sequences on  $\Sigma$ . For the general case, we obtain a recurrence formula describing the frequency of a given word. Then we use this formula to give a precise description of the frequencies of factors in a circular marked uniform D0L word.

*Keywords:* D0L words, substitution dynamical systems.

### 1. Introduction

We study the frequency of factors in a D0L word  $w$ , which is a fixed point of a morphism  $\varphi$ . For the general case, we obtain a recurrence formula for a frequency of a given factor; if the D0L word fits some requirements which are satisfied, for instance, if  $\varphi$  is primitive, then this formula makes it possible to calculate the frequency of every factor of  $w$ .

Furthermore, we consider a special case of circular marked uniform D0L words; this class of D0L words contains some famous examples. We obtain a complete description of the set of frequencies of factors in a circular marked uniform D0L word. That is, the set of factors of each length can be partitioned into such groups that all the words in each group have the same frequency. We give the formulas for the cardinality of each of these groups and for the frequency of its elements.

Previously, M. DEKKING [4] obtained such a description for two famous examples of D0L words, namely the THUE-MORSE word (which is uniform, marked, and circular), and the FIBONACCI word (which is neither uniform nor marked).

The set of the frequencies of factors in a D0L word  $w$  can be considered also as a shift invariant probability measure associated with  $w$ . This measure was studied by M. QUEFFÉLEC in [11]; however, the algorithm of computing the values of this measure (i. e., the frequencies of factors) offered in this book is rather complicated.

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