RECOGNIZABILITY OF RECTANGULAR PICTURES BY WANG SYSTEMS [1]

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ABSTRACT

In this paper we introduce labelled WANG tiles (LWT), that are a generalization of Wang tiles defined in [4]. We use LWT to introduce Wang systems (WS), a new formalism to recognize picture languages. We prove that the family of picture languages defined by Wang systems coincides with the family of picture languages recognized by tiling systems defined in [7]. We also consider unary picture languages, i. e. picture languages over an alphabet of one symbol, and define a natural bijection between pictures and elements of $\{a\}^* \times \{b\}^*$. Then we prove that any rational relation of $\{a\}^* \times \{b\}^*$ can be obtained as the image of a recognizable picture language. Finally, we prove that the family of star-free (unary) picture language is included in $\mathcal{R}at(\{a\}^* \times \{b\}^*)$.

Keywords: automata, picture languages, tilings.

1. Introduction

In the last years many papers have been devoted to the study of picture languages [8, 10, 11, 12, 13, 15, 16], that is sets of rectangular arrays of symbols. Many formalisms have been developed to generalize the notion of regular language to the case of picture languages.

In [7] one introduces the notion of *local picture language* using *tiling systems*, then *recognizable picture languages* are defined as *projections* of local picture languages. This definition seems a good generalization, since, in the one dimensional case, regular languages are exactly the projections of local languages (cf. [5]). Nevertheless, such a definition does not give a model of computation similar to finite automata, as in the case of regular languages.

Recognizable picture languages can be characterized using an *existential monadic second order logic* [15, 8]. Another characterization comes from using particular cellular automata called *on-line tessellation automata* [10, 11].

In [12] the notion of *hv-local picture language* is given using *domino systems* instead of tiling systems; then recognizable picture languages are characterized as projections of hv-local picture languages.