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MINIMAL INITIALIZING WORD: A CONTRIBUTION TO ČERNÝ'S CONJECTURE

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ABSTRACT

Černý's conjecture concerning the minimal length of an initializing word of a finite automaton is treated for a class of automata that lies "between" the general case and the example given by Černý. The automata considered are called Černý-like. Within this context they are characterized by permutation groups. For every finite automaton there exists a non-trivial Černý-like automaton as a monomorphic image of the given one. For each number of states the conjecture is proven for two subclasses of the Černý-like automata.

Keywords: finite state machine, initializable, directable, synchronisable automaton, minimal length of an initializing word, Černý's conjecture.

1. Introduction

1964 Černý conjectured that the minimal length of an initializing word of inputs of a finite automaton with m states (without regard of the outputs) does not exceed $(m-1)^2$ if such a word exists. Černý proved the conjecture for $m \leq 5$ [1, 2]. The best proven bounds in the general case are of order m^3 [16, 2, 8, 10, 15]. Some authors deal with finding a minimal initializing word but they give no bound for its length [12, 14]. In [7] a bound and an algorithm for a special class of circuits are considered. Generalisations of the conjecture are treated in [11, 15]. Rystsov also treats the space complexity in searching a minimal word.

With state set $Z := \{1, 2, \ldots, m\}$, the two input functions $p_c := (1\,2\,\ldots\,m)$ (cycle notation for a permutation) and s_c with $s_c m := 1$ and $s_c z := z$, $z \neq m$ Černý defines a finite automaton $\mathbf{C}_m := (Z, \{p_c, s_c\})$ (see Figure 1a) having the minimal initializing word $(s_c p_c^{m-1})^{m-2} s_c$ (to be read from right to left) with length $(m-1)^2$ [1]. Thus the conjectured bound is optimal. We call $\mathbf{C}_m$ the *Černý-automaton*, it's minimal initializing word is the *Černý-word*.

In general the conjecture is not proven yet, but it holds for the following classes of finite automata with proven bounds:

- Circular, initializable automata [8]. (One of the input functions is a circular permutation of all the states, another one is non-injective. m is a prime, *bound* $= (m-1)^2$.)