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A NOTE ON HAMILTONIAN CYCLES IN LEXICOGRAPHICAL PRODUCTS

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ABSTRACT

A typical sufficient condition for the existence of a hamiltonian cycle in a lexicographical product $G[H]$ of two graphs G and H forces G to contain a hamiltonian cycle or G to contain a hamiltonian path and H to have some additional properties. We present some sufficient conditions in terms of toughness and factors which are much weaker in many cases. A typical statement is that $G[H]$ is hamiltonian if G is 2-edge connected and cubic and $|H| \geq 2$.

Keywords: hamiltonian path, lexicographical product, vertex transitive graph, toughness, multiple of a multigraph.

The *lexicographical product* $G[H]$ of two graphs G and H is defined by $V(G[H]) := V(G) \times V(H)$ and $E(G[H]) := \{[(g,h),(g',h')] : [g,g'] \in E(G) \text{ or } g = g' \wedge [h,h'] \in E(H)\}$. Traceablity properties of lexicographical products have been studied in [4] and are in the scope of our interest in the light of two old well-known conjectures.

The first one is due to CHVÁTAL and known as *t-tough-conjecture.* A graph is called *t-tough*, if each separating vertex set S of G satisfies $|S| \geq t \cdot \omega(G-S)$, where $\omega(G-S)$ denotes the number of components of the graph $G - S$. CHVÁTAL conjectured that there exists a t such that any t-tough graph is hamiltonian. Examples show that $t = 2$ would be sharp [2].

The following is simply a restriction of the 2-tough conjecture to nontrivial lexicographical products.

Conjecture 1 *If $G[H]$ is 2-tough and $|H| \geq 2$ then $G[H]$ is hamiltonian.*

Though it is not true that $G[H]$ is 2-tough if G is 1-tough and $|H| \geq 2$, it could be possible that under the latter conditions $G[H]$ is hamiltonian.

The second conjecture is due to LOVÁSZ and states that any connected vertex transitive graph contains a hamiltonian path. This has been verified for certain vertex numbers as p^k, $2p$, $3p$, $4p$, $5p$ and $2p^2$, p a prime number (for a survey see [1]). By now,

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