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ON VALVE ADJUSTMENTS THAT INTERRUPT ALL s-t-PATHS IN A DIGRAPH

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ABSTRACT

When searching for a path in a digraph, usually the following situation is given: Each node v may be entered via an arbitrary incoming arc (u, v), and v may be left via an arbitrary outgoing arc (v, w).

This paper, however, is addressed to graphs with valve nodes, and these nodes cannot arbitrarily be entered and left. More precisely, a movable valve is installed in each valve node v. Entering v via (u, v) and leaving it via (v, w) is only possible if the current position of the valve generates a connection between these two arcs; if, however, the current valve adjustment interrupts this connection then every path using the arcs (u, v) and (v, w) is interrupted, too.

We investigate the complexity of the following problem:

> Given a digraph with valve nodes. Let s and t be two nodes of this graph. Does there exist a valve adjustment that interrupts all paths from s to t?

We show that this problem can be solved in deterministic polynomial time if all valve nodes belong to a particular class of valves; otherwise, the problem is $\mathcal{NP}$-complete.

Keywords: graph theory, paths in graphs, complexity theory, $\mathcal{NP}$-completeness.

1. Introduction

One of the most prominent problems in graph theory is the search for paths in digraphs. Usually, the following assumption is made: A path may use any incoming arc when entering a node, and a path may use any outgoing arc when leaving a node. This paper, however, is about the situation that a path may not use arbitrary arcs. More precisely, we assume that valves are installed in several nodes of the given network; a path may only use arcs that are connected by the current valve adjustment. An example is given in Figure 1. The valve adjustment on the left side of this figure has the consequence that only the arcs r_1^+ and r_2^+ may be used immediately after r_1^-; the arc r_3^+ is forbidden in this case. The valve adjustment in the middle of Figure 1 has the consequence that only the arc r_2^+ may be used immediately after r_2^-. The valve adjustment on the right side of Figure 1 has the consequence that only the arc r_3^+ may be used immediately after r_3^-.