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CONVEX ALGEBRAS, CONVEX MODULES AND FORMAL POWER SERIES ON TREES

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ABSTRACT

We obtain behaviors of stochastic treeautomata by means of convex algebras. The following minimal realization result is established: for each convexly reachable algebra $\mathcal{A}$, there exists a unique convex epimorphism $\mathcal{A} \to \mathrm{CSynt}(S_{\mathcal{A}})$, where $\mathrm{CSynt}(S_{\mathcal{A}})$ denotes the syntactical convex algebra associated with the treeseries $S_{\mathcal{A}}$ computed by $\mathcal{A}$. Treeseries whose syntactical convex algebra is convexly finite, are studied. Finally, the following conditions are shown to be equivalent: i) $S : T_\Sigma \to [0,1]$ is computed by a convexly finite Σ-module, ii) all right derivatives of S are included into the same convexly finite sub-Σ-module of $[0,1]^{T_\Sigma}$, iii) there exist a function $\psi : \Sigma_0 \to [0,1]^n$, a monoid morphism $\phi : P_\Sigma \to \mathrm{Stoch}_{n\times n}$ compatible with ϕ and a vector $\gamma \in \mathrm{Stoch}_n$, so that $(S,t) = \psi(c)\phi(\tau)\gamma$ for $t = c\tau$ where $\mathrm{Stoch}_{n\times n}$ and Stoch_n denote the monoid of stochastic $n \times n$ matrices and stochastic n-vectors, respectively.

Keywords: convex set, convex algebra, convex module, stochstic treeseries.

0. Introduction

Stochastic treeautomata are $\mathbb{R}$-linearized bottom up treeautomata verifying the following condition: for any string of states and any input letter of suitable rank, the next-move vector is stochastic.

The interest in considering such machines comes from the fact that TURAKAINEN's theorem still holds in this case (i. e. the "positive parts" of recognizable treeseries coincide with the forests $\{t \mid (f,t) > \lambda\}$, where $f : T_\Sigma \to [0,1]$ is the behavior of a double stochastic treeautomaton and λ is a cut point).

This allows to get more information about the structure of recognizable treeseries and to study more in depth the family of stochastic languages (cf. [4, 5]).

Our aim in the present work is to connect stochastic recognition with convex algebraic structures; this will permit us to establish nice minimal realization results.

The paper is divided into five sections; after a preliminary discussion on convex sets and maps (section 1) we show that behaviors of stochastic treeautomata can be computed by means of convex algebras that are convex sets endowed with multiconvex operations.